Overview of Multi-Antenna Regenerative Cooperative Relay Network in Rayleigh Fading Channel

Abstract — Performance of cooperative relaying employing infrastructure based fixed relays having multiple antennas has been investigated. The next generation wireless systems are supposed to handle high data rate as well as large coverage area. It should be consume less power and utilize bandwidth efficiently. At the same time, the mobile terminals must be simple, cheap, and smaller in size. Diversity can be achieved with the help of multi-antenna system where multiple antennas are installed at transmitter/receiver. In this paper we will study regenerative, non regenerative modes, multi-antenna, and radio spectrum for combating multipath fading channel of cooperative relay wireless communication system.

Index Terms — Multi-antenna relay, Regenerative mode, Rayleigh fading channel

I. INTRODUCTION

Spatial diversity has been widely exploited in wireless communication system to combat multipath fading. A new technique called cooperative relaying [1], in which spatially distributed nodes cooperate to improve the quality of communication between two nodes, is emerging as a topic of growing interest. In this system multiple nodes share their antennas and create virtual antenna array [2]. In [3], closed form expression for outage probability in dissimilar Rayleigh fading has been presented, here communication between source and destination is supported by multiple single antenna relay. Analysis of outage probability of selection cooperation of different signal-to-noise ratio (SNR) has been given in [4]. In recent years multiple antennas at relay nodes has been introduced as a promising technique by the research community. Employment of multiple antennas on mobile user nodes is not an attractive solution due to space, complexity and cost constraints, however, installation of multiple antennas at infrastructure based fixed relay nodes is an interesting solution which is gaining high attention. Approximate error analysis of such system model is analyzed in [5]. Here, communication between source and destination is supported by multiple antenna relay nodes. In [6], the authors consider the cellular uplink communication, in which the source node is equipped with multiple antennas [7]. For this case, the authors analyzed the relay and destination performs MRC of the signals in Rayleigh fading channel.

II. Multiple Input & Multiple Output Antenna

The multiple antennas can be used to increase data rates through multiplexing or to improve performance diversity. In MIMO system the transmit and receives antennas can both be used for diversity gain. The basic idea behind MIMO [5] is to exploit the space resource of the propagation channel and combine it with sophisticated signal processing to achieve significant gain in spectral efficiency.

A. System, Channel, and Signal Models

We consider a wireless link equipped with $L_t$ antenna elements at the transmitter and $L_r$ antenna elements at the receiver[10] as shown in fig.2. The discrete equivalent $L_r \times 1$ received vector $\mathbf{r}$ can modeled as

$$\mathbf{r} = \mathbf{Hd} \mathbf{w_t} \mathbf{s_d} + \mathbf{n}$$

Where $\mathbf{s_d}$ is the transmitting signal of the desired user and $\mathbf{n}$ is AWGN vector with zero mean and covariance matrix $N_0 \times L_r$. Without loss of generality, we assume that $\mathbf{s_d}$ has unit average power. $\mathbf{w_t}$ represents the weight vector at the transmitter with $||\mathbf{w_t}||^2 = E_t$. In [5], $\mathbf{n}$ is modeled as

$$\mathbf{n} = \sqrt{N_0} \mathbf{w_n}$$

Where $\mathbf{w_n}$ is the noise vector with zero mean and covariance matrix $N_0 \times L_r$. Without loss of generality, we assume that $\mathbf{n}$ has unit average power. $\mathbf{w_t}$ represents the weight vector at the transmitter with $||\mathbf{w_t}||^2 = E_t$.
B. Optimum weight vector and output SNR

The optimum combining vector at the receiver is well known to given by

\[ \mathbf{w}_r = \mathbf{c} \mathbf{H}_D \mathbf{w}_t \]

where \( \mathbf{c} \) is a constant that does not affect the output SNR, the resulting condition (on \( \mathbf{w}_t \)) maximum SNR is given by

\[ \mathbf{F} = \mathbf{H}_D \mathbf{H}_D^H \]

It is obvious that \( \mathbf{F} \) is a Hermitian nonnegative definite matrix. Therefore the maximum output [9] SNR is given by

\[ \gamma = \frac{\text{E}[s]}{\text{N}_0} \lambda_{\text{max}} \]

where \( \lambda_{\text{max}} \) is the largest eigenvalue of the matrix \( \mathbf{H}_D \mathbf{H}_D^H \), or equivalently, the largest eigenvalue of \( \mathbf{H}_D^H \mathbf{H}_D \). Note that \( \lambda_{\text{max}} > 0 \) with probability 1; therefore the maximum output SNR \( \gamma \) defined is always positive, as expected. Note that the signals are transmitted along the direction of the eigenvector corresponding to the largest eigenvalue of \( \mathbf{H}_D^H \mathbf{H}_D \), and as such the MIMO MRC scheme is also sometimes referred to as “beamforming.”

III. RAYLEIGH FADING

The Rayleigh fading distribution is frequently used to model multipath fading with no direct line of sight path. In this case the channel fading amplitude \( \alpha \) is distributed according [11]

9.15.2 Optimum Weight Vectors and Output SNR

Before analyzing the performance of the MIMO system of interest, we review the MIMO MRC combining scheme considered in Refs. 231, 232, 233. The optimum combining vector at the receiver (given the transmitting weight vector \( \mathbf{w}_t \)) is well known to be given by

\[ w_r = \mathbf{c} \mathbf{H}_D w_t \]

(9.751)

where \( \mathbf{c} \) is a constant that does not affect the output SNR. The resulting conditional (on \( \mathbf{w}_t \)) maximum SNR is given by

\[ \gamma = \frac{1}{\text{N}_0} \frac{\mathbf{w}_t^H \mathbf{H}_D^H \mathbf{H}_D \mathbf{w}_t}{\text{N}_0} \]

(9.752)

According to the Rayleigh–Ritz theorem [235, Sect. 4.2.2], for any nonzero \( N \times 1 \) complex vector \( \mathbf{x} \) and a given \( N \times N \) Hermitian matrix \( \mathbf{A} \), \( \mathbf{x}^H \mathbf{A} \mathbf{x} \leq \| \mathbf{x} \|^2 \lambda_{\text{max}} \).
2.2.1 Multipath Fading

Multipath fading is due to the constructive and destructive combination of randomly delayed, reflected, scattered, and diffracted signal components. This type of fading is relatively fast and is therefore responsible for the short-term signal variations. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multipath fading envelope.

2.2.1.1 Rayleigh

The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. In this case, the channel fading amplitude $\alpha$ is distributed according to:

$$p_{\alpha}(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right), \quad \alpha \geq 0$$

(2.6)

1. Our performance evaluation of digital communications over fading channels will generally be a function of the average SNR per symbol $\gamma$.

The instantaneous SNR per symbol of the channel $\gamma$ is distributed according to an exponential distribution given by:

$$p_{\gamma}(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0$$

(2.7)

and hence, following (2.3), the instantaneous SNR per symbol of the channel $\gamma$ is distributed according to an exponential distribution given by:

The MGF corresponding to this fading model is given by:

$$M_{\gamma}(s) = (1 - s\bar{\gamma})^{-1}$$

(2.8)

In addition, the moments associated with this fading model can be expressed by:

$$E(\gamma^k) = \Gamma(1 + k) \bar{\gamma}^k$$

(2.9)

where $\Gamma(\cdot)$ is the gamma function. The Rayleigh fading model therefore has an AF equal to 1, and typically agrees very well with experimental data for mobile systems where no LOS path exists between the transmitter and receiver antennas [3]. It also applies to the propagation of reflected and refracted paths through the troposphere [7] and ionosphere [8,9], and to ship-to-ship [10] radio links.

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Then the channel Rayleigh fading and joint pdf equation gives the

\[ p \left( \{r(t)\}, \{\alpha(t)\} \right) = K \prod_{l=1}^{L} \sum_{m=1}^{M} \exp \left( -\frac{a_l^2}{N_l} \right) \left( \frac{\alpha_l^2}{\Omega^2} \right) \phi \left( \frac{\alpha_l^2}{\Omega^2} \right) \]  

(7.13)

We now consider the evaluation of (7.13) for Rayleigh and Nakagami-m fading.

### 7.2.1 Rayleigh Fading

For Rayleigh fading with channel PDFs

\[ p_\alpha (\alpha_l) = \frac{a_l^2}{\Omega^2} \exp \left( -\frac{a_l^2}{2\Omega^2} \right), \quad \alpha_l \geq 0 \]  

(7.14)

and \( \Omega^2 = E [\alpha_l^2] \), the integrals of (7.13) can be evaluated in closed form. In particular, using [7], Eq. (3.462.5), p. 382, we obtain

\[ p \left( \{r(t)\}, \{\alpha(t)\} \right) = E \prod_{l=1}^{L} \left( 1 + \frac{a_l^2}{N_l} \right)^{-\frac{1}{2}} \exp \left( -\frac{1}{4} \sqrt{\pi} U_{\alpha} \exp \left( \frac{U_{\alpha}^2}{4} \right) \left( 1 - Q \left( \frac{U_{\alpha}}{\sqrt{2}} \right) \right) \right) \]  

(7.15)

where \( Q(x) \) is the Gaussian Q-function (see Chapter 4), \( \bar{U}_{\alpha} = E_{l} E_{s}/N_{l} \) is the average SNR of the \( i \)th signal over the \( l \)th path, and

\[ U_{\alpha} = \sqrt{\frac{E_s}{N_l} \left( 1 + \frac{1}{U_{\alpha}} \right) \left( 1 - Q \left( \frac{U_{\alpha}}{\sqrt{2}} \right) \right)} \]  

(7.16)

The combination of (7.15) and (7.16) agrees, after a number of corrections, with the results of Eq. (2.8) in Ref. 3 using a different notation.

The decision metric analogous to (7.8) is obtained by taking the natural logarithm.

Where \( Q(x) \) is the Gaussian Q-function and the average SNR[10] of the \( i \)th signal over the \( l \)th path.
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where $Q(x)$ is the Gaussian Q-function (see Chapter 4), $\bar{y}_k = \bar{\gamma}_k E_k / N_0$ is the average SNR of the $k$th signal over the $t$th path, and

$$ U_k \triangleq \frac{E_k}{N_0} \left( \frac{\bar{y}_k}{1 + \bar{y}_k} \right) \left[ \frac{1}{E_k} \Re \left[ e^{-j\theta_k (\gamma)} \right] \right] $$

(7.16)

The combination of (7.15) and (7.16) agrees, after a number of corrections, with the results of Eq. (28) in Ref. 3 using a different notation.

The decision metric analogous to (7.8) is obtained by taking the natural logarithm of (7.15) and ignoring the $\ln K$ term, which results in

$$ A_K = -\frac{1}{2} \sum_{l=1}^{L} \ln (1 + \bar{y}_l) + \frac{1}{2} \sum_{l=1}^{L} \ln \left( 1 + \sqrt{E_k} \exp \left( \frac{\bar{y}_l E_k}{4} \right) \right) $$

(7.17)

The first summation in (7.17) is a bias, and the second summation is the decision variable that depends on the observation. For large average SNR (i.e., $\bar{y}_k \gg 1$), the decision metric above simplifies to (ignoring the $\ln \sqrt{E_k}$ term)

$$ A_K = -\frac{1}{2} \sum_{l=1}^{L} \ln \bar{y}_l + \frac{1}{2} \sum_{l=1}^{L} \ln \left( 1 + \frac{\bar{y}_l E_k}{4} \right) $$

(7.18)

evaluated in closed form. In particular, using Eq. (6.634-4) of Ref. 7 (on p. 739), we obtain after some manipulation

$$ p \left( \{ \gamma_k (t) \}_{k=1}^{K} | \{ y_{\ell} (t) \}_{\ell=1}^{L} \right) = K \prod_{k=1}^{K} (1 + \bar{y}_k)^{-1} \exp \left( \frac{(U_k')^2}{4} \right) $$

(7.28)

where, analogous to (7.16) for the coherent case

$$ U_k' \triangleq \frac{E_k}{N_0} \left( \frac{\bar{y}_k}{1 + \bar{y}_k} \right) \left[ \frac{1}{E_k} \Re \left[ e^{-j\theta_k (\gamma)} \right] \right] $$

(7.29)

Once again taking the natural logarithm of the likelihood of (7.28) and ignoring the $\ln K$ term, we obtain the decision metric

$$ A_K = -\frac{1}{2} \sum_{l=1}^{L} \ln (1 + \bar{y}_l) + \frac{1}{2} \sum_{l=1}^{L} \frac{E_k}{4 N_0} \left( \frac{\bar{y}_l E_k}{1 + \bar{y}_k} \right) \left[ \frac{1}{E_k} \Re \left[ e^{-j\theta_k (\gamma)} \right] \right] $$

(7.30)

A receiver that implements a decision rule based on the metric of (7.30) is illustrated in Fig. 7.4.

For the special case of constant envelope signal sets, wherein the bias [first term of (7.30)] becomes independent of $k$ and can be ignored, the decision metric becomes (ignoring the scaling by the energy $E_k$)
For the special case of constant signal states, wherein the bias is independent of $k$ and can be ignored, the decision metric becomes

\[ \Lambda_{l} = \sum_{i=1}^{P} \ln \left( 1 + \frac{\gamma_i}{\bar{\gamma}_i} \right) + \frac{1}{4N_l} \left( \frac{\bar{\gamma}_0}{1 + \bar{\gamma}_0} \right) \left( \frac{1}{E} |\bar{\gamma}(0)|^2 \right)^2 \]  

(7.30)

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\[ \Lambda_{l} = \sum_{i=1}^{P} \left( \frac{\bar{\gamma}_0}{1 + \bar{\gamma}_0} \right) |\bar{\gamma}(0)|^2 \]  

(7.31)

where $\bar{\gamma}_0 = \Omega_0 / N_0$. If we further assume $N_l = N_0$, $l = 1, 2, \ldots, L_p$, then (7.31) simplifies further to (ignoring the scaling by $N_0$)

\[ \Lambda_{l} = \sum_{i=1}^{P} \left( \frac{\bar{\gamma}_0}{1 + \bar{\gamma}_0} \right) |\bar{\gamma}(0)|^2 \]  

(7.32)

Finally, for a flat power delay profile (PDP), $\Omega_0 = \Omega_0$, $l = 1, 2, \ldots, L_p$, then ignoring the scaling by $\bar{\gamma}(1 + \bar{\gamma})$, the decision metric is simply

\[ \Lambda_{l} = \sum_{i=1}^{P} |\bar{\gamma}(0)|^2 \]  

(7.33)

which is identical in structure to the optimum receiver for a pure AWGN multi-channel, that is, each finger implements a complex crosscorrelator matched to the
A-\text{AVERAGE SNR FOR RAYLEIGH FADING}

Finally, for a flat power delay profile (PDP), $\Omega_j = \Omega_j$; $j = 1, 2, \ldots, L_p$, then ignoring the scaling by $\gamma/(1 + \gamma)$, the decision metric is simply

$$\Lambda_0 = \sum_{j=1}^{L_p} \left( \frac{1}{1 + \gamma_j} \right) |\gamma_j(\tau_j)|^2$$

which is identical in structure to the optimum receiver for a pure AWGN multichannel, that is, each finger implements a complex crosscorrelator matched to the

\[\text{Average BEP performance}\]
\[\text{for optimum reception of noncoherently detected binary FSK over Rayleigh fading with exponential PDP: m=1, M=2n+6, 6n(b_0) = 0.5}\]
IV. CONCLUSION

Performance of cooperative relay schemes employing infrastructure based fixed relay having multiple antennas, has been investigated. Analytical expression of outage probability, BER and throughput, which depend upon the integer value of m. Results show that, number of antennas on the relay, its location, transmission rate and combining schemes at relay destination play a vital role on the system performance.

REFERENCES: